In my opinion, if there is one area of mathematics you should really get good at, it would be vector calculus. Learning vector calculus will build a strong basis for understanding any area of physics and also many advanced areas of math. However, with the vast amount of information available, most people simply do not know where to start learning vector calculus, even though they would like to do so. This article aims to fix that. My goal is to give you a complete step-by-step guide to learn vector calculus without you having to do extensive research on which books to get and where to start. Now, these steps and tips are aimed at people who perhaps don't have a technical background in physics or math or who are trying to learn vector calculus on their own (without academic education). So, if you're just getting started on vector calculus, the steps and tips given in this article should really benefit you as long as you follow them, of course! Basically, vector calculus is the study of applying basic calculus concepts (such as derivatives and integrals) to vectors. Vector calculus also equivalently goes by the name of multivariable calculus, as these are pretty much the same area of mathematics. Before learning vector calculus, you should have a solid understanding of single-variable calculus. You should also learn basic vector operations like addition and dot products as well as the basics of analytic geometry, which involves using coordinate systems to represent various geometric concepts. Now, there are only a couple prerequisites you absolutely need to get by when learning vector calculus. These essential prerequisites are: Single-variable calculus Basic vector operations and the geometry of these Basic analytic geometry In my opinion, you'll be able to learn vector calculus quite well with just these. You'll find more on specifically what you need to know from these areas and where you can learn these later in this article. However, there are also a few things that will definitely be helpful to learn as well, but not strictly necessary. These more so optional prerequisites are: Basic linear algebra Ordinary differential equations Next, let's actually take a look at the practical steps you need to take to learn all these prerequisites and then what to do to learn vector calculus. Down below are my recommended steps for learning vector calculus and which topics you'll want to focus on learning. These steps are either what I've done myself that have personally been beneficial to me or what I would do in case I was to start learning vector calculus from scratch. Later in the article, I'll also explain everything you need to know, what to expect on your journey to master vector calculus as well as my recommended resources for learning more about each topic. Anyway, here are the steps you should take to learn vector calculus, at a very high level: Learn basic vector algebra and geometry. Learn single-variable calculus. Learn the basics of linear algebra (not absolutely necessary, but it'll help). Study ordinary differential equations (not absolutely necessary again, but very helpful). Get familiar with the basics of multivariable calculus. Study vector calculus from a resource dedicated on it. Next, I'll explain each of these steps in more detail as well as give you some resources (both written and video lecture focused material). Vector calculus, as the name may suggest, is based on the concept of vectors. These are geometric objects that are used to encode lengths and directions (although there are also many more advanced definitions). Therefore, learning to work with vectors is essential regarding vector calculus. In vector calculus, you're going to be basically combining calculus with vectors and geometry. I've found that most resources on vector calculus do indeed cover basic vector algebra also, but it helps a lot to be familiar with these basics first. For basic vector algebra, you should understand at least the following concepts before learning anything else: Basic vector operations (addition, dot product, cross product) and the geometry behind these The geometry of a vector (its magnitude, direction and representing vectors as "arrows" in coordinate systems) Working with vector components (how to use these to describe properties of the vector, such as calculate its length or the angle between two vectors) I find it also helps a lot to understand the role of vectors in physics (how physical quantities, like velocity, are described by vectors) Now, something that goes hand-in-hand with the above topics is learning basic analytic geometry (which is why it's included in this same step). Analytic geometry is basically doing geometry in a coordinate system. For example, in analytic geometry, we can describe a circle as an equation involving the Cartesian (x,y) -coordinates. Understanding analytic geometry is essential for vector calculus since in vector calculus, you're going to be working with coordinate systems a lot. That said, from the area of analytic geometry, you should at least learn the following: Working with Cartesian coordinates (x,y,z) and the concepts of lengths, points, angles and basis vectors How to describe geometries like lines, circles and spheres in Cartesian coordinates Using vectors to describe various geometric concepts (such as describing areas by cross products) The basics of working with polar, spherical and cylindrical coordinates For learning all of these topics, I'd recommend the following: Advanced Math For Physics: A Complete Self-Study Course (link to a page with more info): this is my own full online course that covers many math and physics related topics, notably topics like like vectors, coordinate systems as well as single-variable calculus. The course includes all of the prerequisites, basics, and the main topic of vector calculus, of course. The course also comes with multiple workbooks with practice problems and solutions. Another important prerequisite to learn before vector calculus is going to be basic single-variable calculus. This is important because vector calculus essentially expands the concepts of single-variable calculus to multiple dimensions and to different coordinate systems. From single-variable calculus, you should at least have a solid understanding of the following topics: Limits and the notion of derivatives Basic derivative rules (like the product and chain rule) as well as the geometry behind a derivative Integrals and the geometry behind integration Basic integration rules and techniques (like u-substitution and integrating trigonometric functions) The fundamental theorem of calculus Some basic applications of single-variable calculus Now, my resource recommendation would again be my own Advanced Math For Physics: A Complete Self-Study Course, since it aims to teach you all of this (and much much more) in a highly practical way. The reason I'm grouping both of these topics into the same step is because I've found that linear algebra and differential equations are not required for understanding vector calculus (unlike single-variable calculus, for example), but they certainly help a lot. I've therefore classified these as optional topics to learn about, however, I would strongly suggest you do take the time to learn these as they will greatly help you deepen your knowledge of vector calculus as well. That being said, from these topics, you should learn (again, not necessarily required but definitely helpful): The concepts of matrices and linear transformations Determinants and inverses of matrices as well as various classes of matrices (such as orthogonal matrices) Basic solution techniques for ordinary first and second order differential equations Basic applications of differential equations (as Newton's second law in physics, for example) The best free resources I've found for these are: Vector calculus and multivariable calculus are, for all practical purposes, synonymous. However, usually when people study multivariable calculus in and of itself, they don't learn about the more advanced concepts that they would come across in vector calculus. So, in that sense, it's worth distinguishing these two; multivariable calculus has to do with differentiating functions of multiple variables and vector calculus then expands on that even more by the use of abstract vector notation. In any case, what I'd recommend you do in this step is to familiarize yourself with basic concepts of multivariable calculus, which include: Working with functions of multiple variables and understanding what they represent geometrically First, second and mixed partial derivatives of multivariable functions Total derivatives and total differentials of multivariable functions Triple and double integrals of multivariable functions With these concepts in your toolbox, you're going to be in a great position to understand pretty much all topics in vector calculus. To learn these, however, I'd recommend checking out my own Advanced Math -course, as this will cover exactly the above topics with the purpose of using these concepts to understand vector calculus. With all of the above stuff you've now learned, you should have absolutely no trouble understanding vector calculus. In this step, you want to find a resource that's fully dedicated to teach you the concepts of vector calculus. You'll want to find something that, at the very least, covers the following topics: Working with vector fields and the geometry behind these The concepts of gradient, directional derivatives, divergence, curl and Laplacian The basic geometry of curves and surfaces (parameterization, tangent vectors and normal vectors) Line integrals, circulation, surface and flux integrals Stokes' theorem and the divergence theorem Some applications to optimization, physics or other areas With the above topics, you have the basic toolbox of vector calculus at your disposal and you can also apply it to various areas or physics or math. Ideally, however, you also want a resource that includes practice problems you can do yourself. You won't truly learn the concepts above without actually solving problems yourself. Also, there are some topics in vector calculus that are incredibly important but are not covered very often (such as the Helmholtz decomposition theorem, which has A LOT of applications in physics). So, if you can find something that covers these also, then fantastic. Personally, for resources that cover everything mentioned above, I would recommend the following: My own Advanced Math For Physics: A Complete Self-Study Course (link to the course page): This course is aimed at teaching you vector calculus at your own pace and without many prerequisites. The course also covers more advanced topics (such as the Helmholtz theorem and basic tensor notation) as well as many physics applications. The course comes with workbook(s) with practice problems and solutions, which allows you to practice the concepts you learn. Now that we've gone through the necessary steps to learning vector calculus, there may still be some questions you're wondering, such as "how difficult will learning vector calculus be?" or "how long will it take to learn these things?". This is what I'll answer next. Keep in mind that these are based on my own experience after actually studying vector calculus myself. I cannot speak for everyone and I'm sure that other people will have different experiences, so these will only be rough outlines or what you should at least be expecting. The most common question pretty much everyone wanting to learn vector calculus will have at some point is; is vector calculus actually hard? If so, how hard? Vector calculus is not hard for most people with a solid understanding of single-variable calculus. This is because vector calculus simply generalizes the concepts of single-variable calculus to multiple dimensions. However, some of the unfamiliar notation used in vector calculus may seem hard at first. Let me elaborate on this a bit more. Essentially, if you know how to use the tools of single-variable calculus (like calculating derivatives and integrals) and you also understand the geometric intuition behind these, vector calculus will simply just expand on the things you already know. However, vector calculus also comes with some new notation that you're going to have to learn and this is where most of the difficulty actually comes from. For example, one of the most common tools used in vector calculus is the gradient operator, which acts on a function and gives you a vector of the following form: \vec{abla}f=\frac{\partial f}{\partial x}\overline{\text{i}}+\frac{\partial f}{\partial y}\overline{\text{j}}+\frac{\partial f}{\partial z}\overline{\text{k}} This "upside down triangle" (called nabla) is used as a shorthand to denote the gradient and pieces of notation like this are what you'll encounter a lot in vector calculus. Also, from personal experience, probably the most difficult thing in vector calculus is going to be to calculate various line and surface integrals as these often require a bit of geometric understanding of the problem. Other than that, learning vector calculus should not be a problem for anyone with the necessary prerequisites discussed in this article. Good learning resources will also get you there a lot faster, which is what I've aimed to help you find through this article and the steps described earlier. When you begin to learn a new topic like vector calculus, you have to set an expectation of how long it should take to learn it all. But how long exactly? It takes about 5 weeks to learn vector calculus if you already have a solid understanding of single-variable calculus and you focus all of your learning towards vector calculus. However, if you're starting from scratch, it may take several months to learn all the prerequisites first. The logic behind this is based on the steps I gave earlier and some estimates from university/college courses. The rough time frame I estimated in the following way: If you already know basic high school math (single-variable calculus, geometry, basic vector stuff) and are confident in your ability to use it, you can jump right into learning multivariable calculus or vector calculus. A standard multivariable/vector calculus course (Calc III) is usually about 12 weeks long. However, if you're taking, for example, the single-variable and vector calculus sections of my Advanced Math For Physics -course, which you can complete at your own pace and you complete about 2-3 lessons per week (and do all the practice problems), you can easily finish the course in 4 weeks. If you're really starting from scratch and learning all the basic high school level math needed (either from the resources suggested earlier or elsewhere) and doing practice problems as well, expect to add another 8-16 weeks to these numbers (and possibly more, depending on your starting point). The time it will take for you personally to learn vector calculus may vary greatly from this minimum of 5 weeks -estimate. I assume it took me about this long, but I had already graduated high school and I had the required knowledge of single-variable calculus and so on. It may take you much much longer if you need to learn all the necessary math before learning vector calculus. However, this should not discourage you because really, everything you learn along the way is a positive even if it's not vector calculus right away. As a bottom line, you should expect that building a solid understanding of vector calculus will be in a timescale of weeks, at the very least. Learning vector calculus will, by no means, require being a student at university or college and you absolutely CAN learn the topic on your own as well if that's something you want to do. Did this video help you?Vectors can be used to prove two lines are parallel (see Vector Addition)They can also be used to show points are collinear (lie on the same straight line)Vectors can be used to find missing vertices of a given shapeYou will need a good understanding of how to divide a line segment into a given ratioWhen problem-solving with vectors, trigonometry can help us:convert between component form and magnitude/direction form (see Magnitude Direction)find the angle between two vectors using Cosine Rule (see Non-Right-Angled Triangles)find the area of a triangle using a variation of Area Formula (see Non-Right-Angled Triangles)Think of vectors like a journey from one place to another – you may have to take a detour eg. A to B might be A to O then O to B.Diagrams can help, if there isn't one, draw one. For a given diagram labelling all known vectors and quantities will help.Did this page help you? We cover both basic theory and applications. In the first week, we learn about scalar and vector fields, in the second week about differentiating fields, in the third week about integrating fields. The fourth week covers the fundamental theorems of vector calculus, including the gradient theorem, the divergence theorem, and Stokes's theorem. These theorems are needed in core engineering subjects such as Electromagnetism and Fluid Mechanics.Instead of Vector Calculus, some universities might call this course Multivariable or Multivariate Calculus or Calculus 3. Two semesters of single variable calculus (differentiation and integration) are a prerequisite.The course is organized into 42 short lecture videos, with a few problems to solve following each video. And after each substantial topic, there is a short practice quiz. Solutions to the problems and practice quizzes can be found in instructor-provided lecture notes. There are a total of four weeks to the course, and at the end of each week, there is an assessed quiz.WHAT YOU WILL LEARNThe dot product and cross productThe gradient, divergence, curl, and LaplacianMultivariable integration, line integrals, flux integrals, cylindrical and spherical coordinatesThe gradient theorem, divergence theorem, and Stokes's theoremSyllabusWEEK 1 – VectorsA vector is a mathematical construct that has both length and direction. We will define vectors and learn how to add and subtract them, and how to multiply them using the scalar and vector products (dot and cross products).We will use vectors to learn some analytical geometry of lines and planes and learn about the Kronecker delta and the Levi-Civita symbol to prove vector identities. The important concepts of scalar and vector fields will be introduced.WEEK 2 – DifferentiationScalar and vector fields can be differentiated. We define the partial derivative and derive the method of least squares as a minimization problem. We learn how to use the chain rule for a function of several variables, and derive the triple product rule used in chemical engineering. From the del differential operator, we define the gradient, divergence, curl, and Laplacian.We learn some useful vector calculus identities and how to derive them using the Kronecker delta and Levi-Civita symbols. Vector identities are then used to derive the electromagnetic wave equation from Maxwell's equation in free space. Electromagnetic waves form the basis for all modern communication technologies.WEEK 3 – Integration and Curvilinear CoordinatesScalar and vector fields can be integrated. We learn about double and triple integrals, and line integrals, and surface integrals. Curvilinear coordinates, namely polar coordinates in two dimensions, and cylindrical and spherical coordinates in three dimensions are used to simplify problems with cylindrical or spherical symmetry. We learn how to change variables in multidimensional integrals using the Jacobian of the transformation.WEEK 4 – Fundamental TheoremsThe fundamental theorem of calculus links integration with differentiation. Here, we learn the related fundamental theorems of vector calculus. These include the gradient theorem, the divergence theorem, and Stokes's theorem.We show how these theorems are used to derive continuity equations, define the divergence and curl in coordinate-free form, and convert the integral version of Maxwell's equations into their more famous differential form.Teacher History of vector calculation: The vector calculus goes back to H. G. Grassmann and parallel to Hamilton. Grassmann published in 1844 the "Lineale Ausdehnungslehre". As a precursor Descartes and Möbius apply. The Irish mathematician William Rowan Hamilton (1805 - 1865) developed the theory of quaternions, which are considered as precursors of the vectors. The term scalar goes back to Hamilton. 1888 the Italian mathematician Giuseppe Peano (1858-1932) developed an axiomatic definition of a vector space. Scalar: Quantities that can be represented by a real number. Examples for scalar values are temperature, mass, ... Vector: Vectors, in addition to its value require the specification of a direction. In physics, e.g. Velocity, field strength, ... In general, a vector is not limited to three dimensions. In general it is an n-tuple of real numbers that is often listed as a column vector. v→= (v1 v2 ⋮ vn) Linear Dependence: Two vectors are called linearly dependent or collinear if one can be converted by multiplication by a scalar in the other. The cross product of collinear vectors disappears. Orthogonality: Two vectors are called orthogonal if the dot product of two vectors vanishes. Geometrically, the vectors are perpendicular to each other then that is the angle enclosed by the vectors is 90°. Unit vector: Vectors of length 1 are called unit vectors. Each vector can be converted by normalizing into the unit vector by the vector is divided by its length. Calculation rules for vectors Multiplication of a vector with a scalar The multiplication of a vector by a scalar positive λ only changes the length of the vector and not direction. In the multiplication with a negative scalar, the direction of the vector changes in the opposite direction. λ·v→= (λ·v1 λ·v2 ⋮ λ·vn) For the multiplication of a vector by a scalar λ the distributive law holds. λ·(v→+w→) = λ·v→+λ·w→ Vectors are added component by component in Cartesian coordinates. The vector addition is commutative and associative. Geometrically, the resulting vector can be constructed by shifting one of the vectors in parallel to the endpoint of the other vector. The connection from the starting point of the first vector to the end point of the second vector is the resulting vector of the vector addition. v→+w→= (v1 v2 ⋮ vn) + (w1 w2 ⋮ wn) = (v1+w1 v2+w2 ⋮ vn+wn) Subtraction of vectors is carried out as by the addition but the components will be subtracted. Geometrically, the construction similar to the addition only the second vector will be mirrored to the opposite direction. v→-w→= (v1 v2 ⋮ vn) - (w1 w2 ⋮ wn) = (v1-w1 v2-w2 ⋮ vn-wn) The scalar product is defined as the product of the components and the sum of these products. The scalar product is not dependent on the sequence (commutative). The name scalar product comes from the fact that the result is a scalar and not a vector. The scalar product can be interpreted geometrically as the product of the projection of one vector in the direction of the other vector. Physically this means that the product is formed only with the component of the vector that is effective in the direction of the other vector. The scalar product can also be expressed geometrically. φ is the angle enclosed by the vectors. v→·w→= (v1 v2 ⋮ vn) · (w1 w2 ⋮ wn) = v1·w1+ v2·w2+ ...+ vn·wn = ∑i=1n(vi·wi) v→·w→ = |v→||w→|cosφ For the scalar product of the distributive law apply u→·(v→+w→) = u→·v→+u→·w→ and the commutative law v→·w→ = w→·v→ The magnitude (amount) of a vector geometrically corresponds to the length of the vector. |v→|= | (v1 v2 ⋮ vn) | = v12+v22+...+vn2 = ∑i=1nvi2 The vector product is defined in three-dimensional Euclidean vector space. The result of the vector product is a vector which is perpendicular to the area spanned by the two vectors and whose length corresponds to the area of the parallelogram. The vector product can also be expressed geometrically. φ is the angle enclosed by the vectors and n the vector perpendicular to the surface. v→×w→= (v1 v2 v3) × (w1 w2 w3) = (v2w3-v3w2 v1w3-v3w1 v1w2-v2w1) v→×w→= |v→||w→|sinφn→ The vector product is anti-commutative v→×w→ = -(w→×v→) The product of a matrix with a vector is a linear image. The multiplication is explained if the number of columns of the matrix is equal to the number of elements of the vector. The result is a vector whose number of components equals the number of rows of the matrix. This means that a matrix with 2 rows always maps a vector to a vector with two components. A·v→= (a11a12...a1m a21a22...a2m ⋮ an1an2...anm)· (v1 v2 ⋮ vm) = (a11v1+a12v2+...+a1mvm a21v1+a22v2+...+a2mvm ⋮ an1v1+an2v2+...+anmvm) Multiple products of vectors are not associative in general. That the general rule is that the order in which the products are carried out is relevant. In general is: u→·(v→·w→)≠(u→·v→)·w→ and u→×(v→×w→)≠(u→×v→)×w→ For the double vector product the development applies: u→×(v→×w→)=v→·(u→·w→)-w→·(u→·v→) The scalar triple product (u→×v→)·w→ is equal to the volume of the plane defined by the three vectors parallelepiped. The scalar triple product is positive if the three vectors form a right hand system. Lagrange's identity: (m→×u→)·(v→×w→)=(m→·v→)(u→·w→)-(u→·v→)(m→·w→) Geometrical approches Equation of the line through the two points P0 und P1 given by the vectors r0 und r1: r→=r0→+λ(r1→-r0→) Distance of two points P0 and P1: |r1→-r0→| Credentials - Imprint - Contact - Home Complex Functions: Linear and quadratic functions, trigonometric and exponential functions plotted with domain colouring method. Complex Functions Linear equations with one and two unknowns, calculator and graphical presentation. Linear Equations Logaritm Sine, Cosine, Tangent Roots: square, cubic, ... Spain France Italy Germany Poland Chinese Credentials - Imprint - Contact - Home \mathrm{diagonalize} \mathrm{eigenvalues} \mathrm{eigenvectors} \mathrm{gauss\:jordan} \mathrm{unit} See All AI explanations are generated using OpenAI technology. AI generated content may present inaccurate or offensive content that does not represent Symbolab's view. Solve vector operations and functions step-by-step Frequently Asked Questions (FAQ) What are vectors in math? In math, a vector is an object that has both a magnitude and a direction. Vectors are often represented by directed line segments, with an initial point and a terminal point. The length of the line segment represents the magnitude of the vector, and the arrowhead pointing in a specific direction represents the direction of the vector. What are the types of vectors? The common types of vectors are cartesian vectors, column vectors, row vectors, unit vectors, and position vectors. How do you add two vectors? To add two vectors, add the corresponding components from each vector. Example: the sum of (1,3) and (2,4) is (1+2,3+4), which is (3,7) vector-calculator en Related Symbolab blog posts AI may present inaccurate or offensive content that does not represent Symbolab's views. View Full Notebook Show Mobile Notice Show All Notes Hide All Notes Mobile Notice You appear to be on a device with a "narrow" screen width (i.e. you are probably on a mobile phone). Due to the nature of the mathematics on this site it is best viewed in landscape mode. If your device is not in landscape mode many of the equations will run off the side of your device (you should be able to scroll/swipe to see them) and some of the menu items will be cut off due to the narrow screen width. Here are a set of practice problems for the Vectors chapter of the Calculus II notes. If you'd like a pdf document containing the solutions the download tab above contains links to pdf's containing the solutions for the full book, chapter and section. At this time, I do not offer pdf's for solutions to individual problems. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution to that problem. Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section. Here is a list of all the sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section. Basic Concepts – In this section we will introduce some common notation for vectors as well as some of the basic concepts about vectors such as the magnitude of a vector and unit vectors. We also illustrate how to find a vector from its starting and end points. Vector Arithmetic – In this section we will discuss the mathematical and geometric interpretation of the sum and difference of two vectors. We also define and give a geometric interpretation for scalar multiplication. We also give some of the basic properties of vector arithmetic and introduce the common \(\i\), \(\j\), \(\k\) notation for vectors. Dot Product – In this section we will define the dot product of two vectors. We give some of the basic properties of dot products and define orthogonal vectors and show how to use the dot product to determine if two vectors are orthogonal. We also discuss finding vector projections and direction cosines in this section. Cross Product – In this section we define the cross product of two vectors and give some of the basic facts and properties of cross products. Here we will learn about more difficult vector problems, including vector routes involving midpoints, fractions and ratios of lengths. We will also look at parallel vectors. There are also vector worksheets based on Edexcel, AQA and OCR exam questions, along with further guidance on where to go next if you're still stuck. Vector problems use vectors to solve a variety of different types of problems. Vectors have both a magnitude and direction and can be used to show a movement. A quantity which has just magnitude (size) is called a scalar. For example, We can write vectors in several ways, Using an arrow Using boldface Underlined \overrightarrow{AB} =\textbf{a}=\underline{a} In order to solve problems involving vectors it is helpful to use several key facts. In order to solve vector problems: Write any information you know onto the diagram. Decide the route. Write the vector. Simplify your answer. Get your free vector problems worksheet of 20+ questions and answers. Includes reasoning and applied questions. DOWNLOAD FREE x Get your free vector problems worksheet of 20+ questions and answers. Includes reasoning and applied questions. DOWNLOAD FREE The shape below is made from 8 equilateral triangles. \overrightarrow{PQ} = \textbf{a},~ \overrightarrow{OQ} = \textbf{b} Find the vector \overrightarrow{QT} . Write any information you know onto the diagram. Parallel vectors of the same magnitude are the same. These triangles are all identical therefore we can label the corresponding vectors a and b . 2Decide the route. We need to find a route where we know the vectors. 3Write the vector. \overrightarrow{QT}=-\textbf{b}-\textbf{b}+\textbf{a}-\textbf{b} 4Simplify your answer. \overrightarrow{QT}=\textbf{a}-3\textbf{b} \overrightarrow{AC}=\textbf{a}, ~\overrightarrow{BC}=\textbf{b} The line AB is extended to the point D so that the length AD is three times the length AB . Find the vector \overrightarrow{AD} . Write any information you know onto the diagram. There is currently no further information we can add to the diagram. We know AD is three times the length of AB . We need to find a route from A to B and then multiply it by three. \overrightarrow{AB}=\textbf{a}-\textbf{b}\\\overrightarrow{AD}=3\overrightarrow{AB}=3(\textbf{a}-\textbf{b}) \overrightarrow{AD}=3\textbf{a}-3\textbf{b} \overrightarrow{BC} = \textbf{p}, ~\overrightarrow{BA}=\textbf{q} D is the midpoint of AC . Find the vector \overrightarrow{BD} . Write any information you know onto the diagram. When we are given information about midpoints, fractions of lines or ratios, it can be helpful to add this information onto the diagram. Here we are going to use the fraction \frac{1}{2} to show that D is half way along the line. We are trying to find a route from B to D . We know that \overrightarrow{BC}=\textbf{p} and we then need to get from C to D . We also know that CD=\frac{1}{2}CA . We can find the vector \overrightarrow{CA} and then half it. \begin{aligned} &\overrightarrow{CA}=-\textbf{p}+\textbf{q}\\\\ &\overrightarrow{CD}=\frac{1}{2}\overrightarrow{CA}=\frac{1}{2}(-\textbf{p}+\textbf{q}) \end{aligned} Now we have a route from B to D . \overrightarrow{BD}=\textbf{p}+\frac{1}{2}(-\textbf{p}+\textbf{q}) \begin{aligned} &\overrightarrow{BD}=\textbf{p}+\frac{1}{2}(-\textbf{p}+\textbf{q})\\\\ &\overrightarrow{BD}=\textbf{p}-\frac{1}{2}\textbf{p}+\frac{1}{2}\textbf{q}\\\\ &\overrightarrow{BD}=\frac{1}{2}\textbf{p}+\frac{1}{2}\textbf{q} \end{aligned} \begin{aligned} &\overrightarrow{AB} = 2\textbf{a}, ~ \overrightarrow{AC} =6\textbf{b}\\\\ &\text{BM }=\frac{1}{4}\text{ BA}, \text{ AN} = \frac{1}{3} \text{ AC} \end{aligned} Find the vector \overrightarrow{MN} . Write any information you know onto the diagram. When we are given information about midpoints, fractions of lines or ratios, it can be helpful to add this information onto the diagram. Here we know \text{BM }=\frac{1}{4}\text{ BA}, \text{ AN} = \frac{1}{3} \text{ AC} . We need to find a route from M to N . We can see that \text{MA}=\frac{3}{4}\text{BA and AN}=\frac{1}{3}\text{AC} . \overrightarrow{MA}=\frac{3}{4}\overrightarrow{BA}=\frac{3}{4}(-2\textbf{a})=-\frac{3}{2}\textbf{a}\\\overrightarrow{AN}=\frac{1}{3}\overrightarrow{AC}=\frac{1}{3}(6\textbf{b})=2\textbf{b} We now have a route from M to N . \overrightarrow{MN}=-\frac{3}{2}\textbf{a}+2\textbf{b} The answer here cannot be simplified. \overrightarrow{FG} =\textbf{a}, ~ \overrightarrow{EH} =2\textbf{a}, ~ \overrightarrow{EF} =\textbf{b} ED =2EH and the point J is such that GJ:JH = 2:1 . Find the vector \overrightarrow{JD} . Write any information you know onto the diagram. When we are given information about midpoints, fractions of lines or ratios, it can be helpful to add this information onto the diagram. Here we know GJ:JH=2:1 . This means that \text{GJ}=\frac{2}{3}\text{GH and JH}=\frac{1}{3}\text{GH} . We also know that ED=2EH therefore HD is the same length as EH and in the same direction. We know \text{JH}=\frac{1}{3}\text{GH} . We can find the vector \overrightarrow{GH} . \begin{aligned} &\overrightarrow{GH}=-\textbf{a}-\textbf{b}+2\textbf{a}=\textbf{a}-\textbf{b}\\\\ &\overrightarrow{JH}=\frac{1}{3}\overrightarrow{GH}=\frac{1}{3}(\textbf{a}-\textbf{b}) \end{aligned} We now have a route from J to D . \overrightarrow{JD}=\frac{1}{3}(\textbf{a}-\textbf{b})+2\textbf{a} \begin{aligned} &\overrightarrow{JD}=\frac{1}{3}(\textbf{a}-\textbf{b})+2\textbf{a}\\\\ &\overrightarrow{JD}=\frac{1}{3}\textbf{a}-\frac{1}{3}\textbf{b}+2\textbf{a}\\\\ &\overrightarrow{JD}=\frac{7}{3}\textbf{a}-\frac{1}{3}\textbf{b} \end{aligned} \overrightarrow{AB} =6\textbf{a}, ~\overrightarrow{BC} =10\textbf{b} E is the point on AB such that \text{EB }=\frac{1}{3}\text{ AB} . D is the point on BC such that CD:DB=1:4 . F is the midpoint of ED . Find the vector \overrightarrow{EF} . Write any information you know onto the diagram. We want to find the the vector \overrightarrow{EF} . We know that \text{EF}=\frac{1}{2}\text{ED} so we can start by finding the vector \overrightarrow{ED} . \begin{aligned} &\overrightarrow{EB}=\frac{1}{3}\overrightarrow{AB}=\frac{1}{3}(6\textbf{a})=2\textbf{a}\\\\ &\overrightarrow{BD}=\frac{4}{5}\overrightarrow{BC}=\frac{4}{5}(10\textbf{b})=8\textbf{b} \end{aligned} \overrightarrow{ED}=2\textbf{a}+8\textbf{b} \overrightarrow{EF}=\frac{1}{2}\overrightarrow{ED}=\frac{1}{2} (2\textbf{a}+8\textbf{b})=\textbf{a}+4\textbf{b} This answer cannot be simplified. Two vectors are parallel if one is a multiple of the other. This is because if one vector is a multiple of another, it is a bigger or smaller version of the other. In order to show two vectors are parallel: Work out each vector. Show that one is a multiple of the other by factorising. Show that AB is parallel to CD . \overrightarrow{CD}=\textbf{a}+2\textbf{b} We need to find the vector \overrightarrow{AB} . \begin{aligned} &\overrightarrow{AB}=4\textbf{a}+\textbf{a}+2\textbf{b}-3\textbf{a}+2\textbf{b}\\\\ &\overrightarrow{AB}=2\textbf{a}+4\textbf{b} \end{aligned} Show that one is a multiple of the other by factorising. \overrightarrow{AB}=2\textbf{a}+4\textbf{b}=2(\textbf{a}+2\textbf{b})=2\overrightarrow{CD} Since the vector \overrightarrow{AB} is a multiple of the vector \overrightarrow{CD} these vectors are parallel. ABCD is a parallelogram. The line AB is extended to the point E such that \text{BE}=\frac{2}{3}\text{AB} . The point F is on the line BC such that BF:FC=2:3 . \overrightarrow{AB}=6\textbf{a}, \overrightarrow{CB}=5\textbf{b} Show that DFE is a straight line. We need to find the vectors \overrightarrow{DF} \text{ and } \overrightarrow{FE} . \begin{aligned} &\overrightarrow{CF}=\frac{3}{5}\overrightarrow{CB}=3\textbf{b}\\\\ &\overrightarrow{FB}=\frac{2}{5}\overrightarrow{CB}=2\textbf{b} \end{aligned} \begin{aligned} &\overrightarrow{DF}=6\textbf{a}+3\textbf{b}\\\\ &\overrightarrow{FE}=4\textbf{a}+2\textbf{b} \end{aligned} Show that one is a multiple of the other by factorising. \[\overrightarrow{\mathrm{DF}}=1.5(4 \mathbf{a}+2 \mathbf{b})=1.5 \overrightarrow{\mathrm{FE}}\] Since \overrightarrow{DF} is a multiple of \overrightarrow{FE} the vectors are parallel. They also share the point F meaning they form a straight line. Remember to make the vector negative when going backwards along it. If two parts of a line are in the ratio 1:4, this means one part is ⅕ of a line and the other part ⅘ . \overrightarrow{CE}=-\textbf{p}+2\textbf{q} -2\textbf{a}+1.5\textbf{b} 2\textbf{a}-1.5\textbf{b} \text{RM}=\frac{1}{2}\text{RQ} \begin{aligned} &\overrightarrow{RQ}=-4\textbf{a}+3\textbf{b}\\\\ &\overrightarrow{RM}=\frac{1}{2}\overrightarrow{RQ}=\frac{1}{2}(-4\textbf{a}+3\textbf{b})=-2\textbf{a}+1.5\textbf{b} \end{aligned} \frac{14}{3}\textbf{a}-\textbf{b} \frac{10}{3}\textbf{a}+\textbf{b} \textbf{b}+\frac{2}{3}\textbf{a} \textbf{b}-\frac{2}{3}\textbf{a} We know \text{DC}=\frac{1}{3}\text{DC} so we need to find the vector \overrightarrow{DC} . \begin{aligned} &\overrightarrow{DC}=-4\textbf{a}+3\textbf{b}+2\textbf{a}=3\textbf{b}-2\textbf{a}\\\\ &\overrightarrow{DE}=\frac{1}{3}\overrightarrow{DC}=\frac{1}{3}(3\textbf{b}-2\textbf{a}) \end{aligned} \begin{aligned} &\overrightarrow{AE}=4\textbf{a}+\frac{1}{3}(3\textbf{b}-2\textbf{a})\\\\ &\overrightarrow{AE}=4\textbf{a}+\textbf{b}-\frac{2}{3}\textbf{a}\\\\ &\overrightarrow{AE}=\frac{10}{3}\textbf{a}+\textbf{b} \end{aligned} \frac{3}{2}\textbf{a}+2\textbf{b} \frac{3}{2}\textbf{a}+\frac{4}{3}\textbf{b} -\frac{3}{2}\textbf{a}-2\textbf{b} \frac{3}{2}\textbf{a}-2\textbf{b} We need to find a route from P to M . We can see \overrightarrow{PB}=6\textbf{b} . We know that BM=\frac{1}{2}BN so we need to find the vector \overrightarrow{BN} . \begin{aligned} &\overrightarrow{BN}=-8\textbf{b}+3\textbf{a}\\\\ &\overrightarrow{BM}=\frac{1}{2}\overrightarrow{BN}=\frac{1}{2}(-8\textbf{b}+3\textbf{a})\\\\ &\overrightarrow{PM}=6\textbf{b}+\frac{1}{2}(-8\textbf{b}+3\textbf{a})\\\\ &\overrightarrow{PM}=6\textbf{b}-4\textbf{b}+\frac{3}{2}\textbf{a}\\\\ &\overrightarrow{PM}=\frac{3}{2}\textbf{a}+2\textbf{b} \end{aligned} \frac{5}{2}\textbf{a}+4\textbf{b} \frac{5}{8}\textbf{a}+\textbf{b} -\frac{5}{4}\textbf{a}-\frac{23}{4}\textbf{b} We need to begin by finding the vector \overrightarrow{AD} . \begin{aligned} &\overrightarrow{AB}=2\textbf{a}+6\textbf{b}\\\\ &\overrightarrow{BC}=-2\textbf{a}-6\textbf{b}+3\textbf{a}+2\textbf{b}=\textbf{a}-4\textbf{b}\\\\ &\overrightarrow{BD}=\frac{1}{2}\overrightarrow{BC}=\frac{1}{2}\textbf{a}-2\textbf{b}\\\\ &\overrightarrow{AD}=2\textbf{a}+6\textbf{b}+\frac{1}{2}\textbf{a}-2\textbf{b}=\frac{5}{2}\textbf{a}+4\textbf{b}\\\\ &\overrightarrow{AE}=\frac{1}{4}(\frac{5}{2}\textbf{a}+4\textbf{b})=\frac{5}{8}\textbf{a}+\textbf{b} \end{aligned} \begin{aligned} &\overrightarrow{AB}=4\textbf{a}\\\\ &\overrightarrow{AC}=4\textbf{a}+4\textbf{b}\\\\ &\overrightarrow{CE}=4\textbf{b}-4\textbf{a}\\\\ &\overrightarrow{BE}=8\textbf{b}-4\textbf{a} \end{aligned} 4\textbf{b}-4\textbf{a}=4(\textbf{b}-\textbf{a}) so \overrightarrow{CE} is parallel to \textbf{b}-\textbf{a} . 1. \overrightarrow{AB}=6\textbf{b},\overrightarrow{BC}=9\textbf{a},\overrightarrow{DF}=-3\textbf{a}+3\textbf{b} F is the midpoint of AB . Show that AD is parallel to BC . (2 marks) \overrightarrow{AD}=3\textbf{b}-(-3\textbf{a}+3\textbf{b})=3\textbf{a} (1) \overrightarrow{BC} =3\overrightarrow{AD} – they are multiples of each other therefore parallel. (1) 2. \overrightarrow{AD}=12\textbf{a}, \overrightarrow{CD}=8\textbf{b} AD=1.5BC. E is the midpoint of AB . EF:FC=1:3. Find the vector \overrightarrow{BF} . (4 marks) \overrightarrow{BC}=8\textbf{a} (1) \overrightarrow{EB}=\frac{1}{2}(12\textbf{a}-8\textbf{b}-8\textbf{a})=2\textbf{a}-4\textbf{b} (1) \overrightarrow{EC}=2\textbf{a}-4\textbf{b}+8\textbf{a}=10\textbf{a}-4\textbf{b} (1) \overrightarrow{EF}=\frac{1}{4}(10\textbf{a}-4\textbf{b})=\frac{5}{2}\textbf{a}-\textbf{b} (1) 3. \overrightarrow{AB}=4\textbf{a}, \overrightarrow{BC}=2\textbf{b} D is the midpoint of AB. Point E is such that AE:EC=3:1 . BF=2BC. Determine whether DEF is a straight line. (4 marks) \overrightarrow{CE}=\frac{1}{4}(-2\textbf{b}-4\textbf{a})=-\frac{1}{2}\textbf{b}-\textbf{a} (1) \overrightarrow{DE}=2\textbf{a}+2\textbf{b}-\frac{1}{2}\textbf{b}-\textbf{a}=\textbf{a}+\frac{3}{2}\textbf{b} (1) \overrightarrow{EF}=\frac{1}{2}\textbf{b}+\textbf{a}+2\textbf{b}=\textbf{a}+\frac{5}{2}\textbf{b} (1) DEF is not a straight line since \overrightarrow{DE} and \overrightarrow{EF} and are not multiples of each other and therefore are not parallel. (1) You have now learned how to: Solve complex problems involving vectors Loci and construction Transformations Circle theorems Prepare your KS4 students for maths GCSEs success with Third Space Learning. Weekly online one to one GCSE maths revision lessons delivered by expert maths tutors. Find out more about our GCSE maths tuition programme. We use essential and non-essential cookies to improve the experience on our website. Please read our Cookies Policy for information on how we use cookies and how to manage or change your cookie settings.AcceptPrivacy & Cookies Policy Part A: Vectors, Determinants and Planes « Previous | Next » Overview In this session you will: Read course notes Review an example Watch a lecture video clip and read board notes Review three additional examples Watch a recitation video Do problems and use solutions to check your work Introduction to Vectors (PDF) Examples Vector Addition (PDF) Lecture Video Clip: Vectors The following images show the chalkboard contents from these video excerpts. Click each image to enlarge. Examples Vector Lengths (PDF) Force is a Vector (PDF) Proofs Using Vectors (PDF) Recitation Video Coordinate Free Proofs: Centroid of a Triangle Problems and Solutions Problems: Vectors (PDF) Solutions (PDF) « Previous | Next » Many students are often reluctant to tackle questions using vectors. I think this is partly because often vectors is not taught until quite a way through a school maths course, so they are unfamiliar. This short article aims to highlight some of the powerful techniques that can be used to solve problems involving vectors, and to encourage you to have a go at such problems to become more familiar with vector properties and applications.So what are vectors? When we first meet them, it's often in the context of transformations - a translation can be expressed as a vector telling us how far something is translated to the right (or left) and up (or down). Confusion can strike when we come across vectors being used to indicate absolute position relative to an origin as well as showing a direction. Then we may be informed that a vector is "simply" a quantity that has both magnitude and direction (unlike a scalar which only has magnitude).DiagramsIt is helpful to separate out some of these ideas about vectors in order to make sense of things. For me, diagrams make it much easier to make sense of what is going on - I can represent a position vector as a point on the diagram with a line segment coming from the origin. Direction vectors just become line segments joined onto other vectors, with a helpful arrow to remind me that $\mathbf{a}$ and $\mathbf{-a}$ are in opposite directions! Sometimes it's useful to draw on lines parallel and perpendicular to my coordinate axes so I can make sense of the x and y components of a vector.Given a vector problem, a quick sketch can help you to see what's going on, and the act of transferring the problem from the written word to a diagram can often give you some insight that will help you to find a solution. Start by solving vector problems in two dimensions - it's easier to draw the diagrams - and then move on to three dimensions. (For four or more dimensions, it becomes more difficult to visualise!)What to do when you get stuckHere is a brief checklist of ideas to think about if you are stuck on a vector question, and drawing a diagram hasn't helped.Parallel vectorsDo you know about any parallel lines? Vector questions can often be about geometrical shapes like trapezia, rhombuses or parallelograms. If two vectors are parallel, it can be really useful to express one in terms of the other - if $\mathbf{a}$ and $\mathbf{b}$ are parallel, try writing $\mathbf{b}=k\mathbf{a}$ for some constant k.Scalar productsScalar products are immensely useful! Sometimes if you're at a loss to know what to do with vectors and vector equations, it's worth just taking the scalar product of the whole equation with one of your vectors and seeing what you end up with. Remember, $\mathbf{a}.\mathbf{a}=|a|^2$, and if two vectors are perpendicular, their scalar product is 0.Magnitude and directionSome vector problems involve a vector function which tells you how an object's position changes in time, for example. Working out how the magnitude and direction change over time can help you to picture the situation.Vector equation of a lineSome students are intimidated by the vector equation of a line when they first meet it. We are very used to expressing lines using cartesian geometry in the form $y=mx+c$ and other variants. The vector equation of a line is no more complicated really, it's just a case of getting used to it. In simple terms, lines are represented using vectors by specifying a point on the line with a position vector, and then using a direction vector to specify the direction of the line. In the same way that $y=mx+c$ specifies a line that passes through $(0,c)$ and has gradient m, the vector equation $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$ specifies a line that passes through the point with position vector $\mathbf{a}$ in the direction of $\mathbf{b}$.A final word on notation; in type, vectors are indicated by bold type. In handwriting, it is a convention to underline vectors and leave scalars (such as the constants k and λ above without underlining. The Greek letters λ and μ are often used as constants in vector equations, so why not get into the habit of using them for yourself? If you've looked into general relativity or differential geometry, you might have... Noether's theorem, named after the German mathematician Emmy Noether, is often said... Most people have heard of basic calculus and know about its applications... Mathematics and Statistics Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. 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