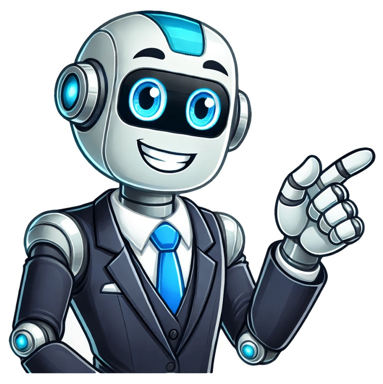

Rectangular to cylindrical coordinates

MathematicsGeometry3D Coordinates Systems Change of 3D Coordinates (space) A 3D coordinate system is a mathematical framework for describing the position of points in three-dimensional space. The main types of 3D coordinate systems are: — Cartesian coordinate system: Uses the $ x $, $ y $ and $ z $ axes to specify the position of a point, each coordinate represents the perpendicular distance of the point from the plane formed by the other two axes. — Cylindrical coordinate system: Uses a radial coordinate $ r $, an angular coordinate $ \theta $, and a height $ z $. Position is determined by the distance $ r $ from a central axis (usually the $ z $ axis), the angle $ \theta $ around this axis, and the height $ z $ along the central axis. — Spherical coordinate system: Uses radial distance $ \rho $, azimuth angle $ \theta $ and colatitude angle $ \varphi $. The position is determined by $ \rho $ the distance from the point to the origin, $ \theta $ is the angle in the $ xy $ plane from the $ x $ axis, and $ \varphi $ is the angle relative to the $ z $ axis. dCode uses the ISO standard for spherical coordinates $ (\rho,\theta,\varphi) $ From Cartesian coordinates $ (x, y, z) $, the base / referential change to spherical coordinates $ (\rho,\theta,\varphi) $ follows the equations: $$ \rho = \sqrt{x^2 + y^2 + z^2} \\ \theta = \arccos \left(\frac{z}{\sqrt{x^2 + y^2 + z^2}} \right) = \arccos \left(\frac{z}{\rho} \right) \\ \varphi = \arctan \left(\frac{y}{x} \right) $$ Example: Le point in space in position $ (0,\sqrt{2},\sqrt{2}) $ from cartesian coordinates is defined by spherical coordinates $ \rho = 1 $, $ \theta = \pi/4 $ and $ \varphi = \pi/2 $ The conversion can be seen as two consecutive Cartesian to Polar coordinates conversions, first one in the $ xy $ plane to convert $ (x, y) $ to $ (R, \varphi) $ (with $ R $ the projection of $ \rho $ on the $ xy $ plane, then a second conversion but in the $ zR $ plane to change $ (z, R) $ to $ (\rho, \theta) $ NB: by convention, the value of $ \rho $ is positive, the value of $ \theta $ is included in the interval $] 0, \pi [$ and the value of $ \varphi $ is included in the interval $] -\pi, \pi [$ If $ \rho = 0 $ then the angles can be defined by any real numbers of the interval From cartesian coordinates $ (x, y, z) $ the base / referential change to cylindrical coordinates $ (r, \theta, z) $ follows the equations: $$ r = \sqrt{x^2 + y^2} \\ \theta = \arctan \left(\frac {y}{x} \right) \\ z = z $$ NB: by convention, the value of $ \rho $ is positive, the value of $ \theta $ is included in the interval $] -\pi, \pi [$ and the $ \varphi $ is a real number From spherical coordinates $ (\rho,\theta,\varphi) $ the base / referential change to cartesian coordinates $ (x, y, z) $ follows the equations: $$ x = \rho \sin\theta \cos\varphi \\ y = \rho \sin\theta \sin\varphi \\ z = \rho \cos\theta $$ From spherical coordinates $ (\rho,\theta,\varphi) $ the base / referential change to cylindrical coordinates $ (r,\theta^*,z) $ follows the equations: $$ r = \rho \sin \theta \\ \theta^* = \varphi \\ z = \rho \cos \theta $$ From cylindrical coordinates $ (r,\theta,z) $ the base / referential change to cartesian coordinates $ (x,y,z) $ follows the equations: $$ x = r \cos\theta \\ y = r \sin\theta \\ z = z $$ From cylindrical coordinates $ (r,\theta^*,z) $ the base / referential change to spherical coordinates $ (\rho,\theta,\varphi) $ follows the equations: $$ \rho = \sqrt{r^2 + z^2} \\ \theta = \arctan \left(\frac{r}{z} \right) = \arccos \left(\frac{z}{\sqrt{r^2 + z^2}} \right) \\ \varphi = \theta^* $$ dCode retains ownership of the "3D Coordinates Systems" source code. 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Exporting the results is free and can be done simply by clicking on the export icons ⤓ (.csv or .txt format) or (copy and paste). To cite dCode.fr on another website, use the link: In a scientific article or book, the recommended bibliographic citation is: 3D Coordinates Systems on dCode.fr [online website], retrieved on 2025-05-24, Summary ▲ Feedback In three-dimensional space, cylindrical coordinates are a logical extension of polar coordinates. A collection of three cylindrical coordinates can be used to identify a point in the cylindrical coordinate system. We can use cartesian and polar coordinates to specify a point's location in two dimensions. An additional z coordinate is introduced when the polar coordinates are expanded to a three-dimensional plane. Together, these three measurements create cylindrical coordinates. Change From Rectangular to Cylindrical Coordinates Recall that a point P in three dimensions is represented by the ordered triple (r, θ, z) in the cylindrical coordinate system. Where r and θ are the polar coordinates of the projection of point P onto the XY-plane and z is the directed distance from the XY-plane to P. Use the following formula to convert rectangular coordinates to cylindrical coordinates. \(r^2 = x^2 + y^2 \) \(tan(θ) = \dfrac{y}{x} \) \(z = z \) Example: Rectangular to Cylindrical Coordinates Let's take an example with rectangular coordinates (3, -3, -7) to find cylindrical coordinates. Substitute the specified ordered triple into the formulae shown above to convert from rectangular to cylindrical coordinates. Remember that the coordinates supplied can be understood as x = 3, y= -3, and r = -7. \(r = \sqrt{x^2 + y^2} \) \(r = \sqrt{(3)^2 + (-3)^2} \) \(r = 3\sqrt{2} \) \(tan(θ) = \dfrac{y}{x} \) \(tan(θ) = \dfrac{-3}{3} \) \(tan (θ) = -1 \) \(θ = tan-1 (-1) \) \(θ = \dfrac{3π}{4} + πn \) \(z = z \) \(z = -7 \) We can write the converted cylindrical coordinates (3, -3, -7) in several forms, such as (\(3\sqrt{2}\), \(\frac{3π}{4}\), -7) and (\(3\sqrt{2}\), \(\frac{7π}{4}\), -7). Key Points on Cylindrical Coordinates A plane's radial distance, azimuthal angle, and height are used to locate a point in the cylindrical coordinate system. These coordinates are ordered triples. The symbol for cylindrical coordinates is (r, θ, z). We can transform spherical and cylindrical coordinates into cartesian coordinates and vice versa. FAQs Are polar and cylindrical coordinates the same? Simple three-dimensional extensions of the two-dimensional polar coordinates are known as cylindrical coordinates. Remember that we can use polar coordinates (r,θ) to define a point's location in a plane. For example, the distance of the point from the origin is represented by the polar coordinate r. What is the radial distance? It is the distance in Euclidean geometry between the point in three-dimensional space and the origin, O (0, 0). Since virtually everything we do in this course deals with three dimensional space, it makes sense to start with a short discussion of how to represent a point in 3-D space. Three dimensional space is often written R3\mathbb{R}^3R3 (read "R three"), to denote that we're dealing with real numbers in three dimensions; similarly, 2-D space is called R2,ℝ^2,R2, the number line is called R,ℝ,R, and n-dimensional space is called Rn.ℝ^n.Rn. We'll cover three ways of describing the location of a point: with rectangular coordinates, cylindrical coordinates, and spherical coordinates. There are other coordinate systems (including some wacky ones like hyperbolic and spheroidal coordinates), but these are the ones that are most commonly used for three dimensions. We won't actually use cylindrical and spherical coordinates for a while, but getting a look at them now can help to get comfortable thinking in three dimensions, and when they come back again, we'll be at least somewhat comfortable with them. As we go through this section, we'll see that in each coordinate system, a point in 3-D space is represented by three coordinates, just like a point in 2-D space is represented by two coordinates (xxx and yyy in rectangular, rrr and θθθ in polar). Rectangular Coordinates Using rectangular coordinates, a point in R3ℝ^3R3 is represented by (x,y,z).(x,y,z).(x,y,z). Similar to what we do in R2,ℝ^2,R2, to get to a specified point, we start at the origin, travel along the xxx axis the distance specified by the first coordinate, then parallel to the yyy axis according to the second coordinate, and then up parallel to the zzz axis according to the third coordinate. We can also talk about the projection of a point (or a line or plane or other figure, later) onto one of the three planes that make up the coordinate system: the xyxyxy plane (where z=0z=0z=0), the yzyzyz plane (where x=0x=0x=0), and the xzxzxz plane (where y=0y=0y=0). To project a point onto any one of these planes, simply set the appropriate coordinate to zero. Cylindrical coordinates are essentially polar coordinates in R3.ℝ^3.R3. Remember, polar coordinates specify the location of a point using the distance from the origin and the angle formed with the positive xxx axis when traveling to that point. Cylindrical coordinates use those those same coordinates, and add zzz for the third dimension. In other words, to find a point (r,θ,z)(r,θ,z)(r,θ,z) in cylindrical coordinates, find the point (r,θ)(r,θ)(r,θ) in the xyxyxy plane, then move straight up (parallel to the zzz axis) according to the third dimension given. For instance, the point (3,π4,4)(3,\dfrac{π}{4},4)(3,4π,4) in cylindrical coordinates is shown below. Converting rectangular coordinates to cylindrical coordinates and vice versa is straightforward, provided you remember how to deal with polar coordinates. To convert from cylindrical coordinates to rectangular, use the following set of formulas: x=rcosθy=rsinθz=z\begin{aligned} x &= r\cos θ\ y &= r\sin θ\ z &= z \end{aligned}xyz=rcosθ=rsinθ=z Notice that the first two are identical to what we use when converting polar coordinates to rectangular, and the third simply says that the zzz coordinates are equal in the two systems. Convert (3,π4,4)(3,\dfrac{π}{4},4)(3,4π,4) in cylindrical coordinates to rectangular coordinates. Solution Use the formulas, noting that r=3,r=3,r=3, θ=π/4,θ=π/4,θ=π/4, and z=4.z=4.z=4. x=rcosθ=3cosπ4=322y=rsinθ=3sinπ4=322z=z=4\begin{aligned} x &= r\cos θ = 3\cos \dfrac{π}{4}=\dfrac{3\sqrt{2}}{2}\ y &= r\sin θ = 3\sin \dfrac{π}{4}=\dfrac{3\sqrt{2}}{2}\ z &= z=4 \end{aligned}xyz=rcosθ=3cos4π=232=rsinθ=3sin4π=232=z=4 Therefore, this point is (322,322,4)\ans{\left(\dfrac{3\sqrt{2}}{2},\dfrac{3\sqrt{2}}{2},4\right)}(232,232,4) in rectangular coordinates. To go in the other direction (from rectangular coordinates to cylindrical), use the following set of formulas (again, the first two are exactly what we use to convert from rectangular to polar in R2ℝ^2R2): r=x2+y2θ=tan−1yxz=z\begin{aligned} r &= \sqrt{x^2+y^2}\ θ &= \tan^{−1}\dfrac{y}{x}\ z &= z \end{aligned}rθz=x2+y2=tan−1xy=z Convert (−2,2,6)(−2,2,6)(−2,2,6) in rectangular coordinates to cylindrical coordinates. Solution Use the formulas, noting that x=−2,x=−2,x=−2, y=2,y=2,y=2, and z=6.z=6.z=6. r=x2+y2=8=22θ=tan−1yx=tan−1(−1)=3π4z=z=6\begin{aligned} r &= \sqrt{x^2+y^2} =\sqrt{8}=2\sqrt{2}\ θ &= \tan^{−1} \dfrac{y}{x}=\tan^{−1}(−1)=\dfrac{3π}{4}\ z &= z=6 \end{aligned}rθz=x2+y2=8=22=tan−1xy=tan−1(−1)=43π=z=6 Therefore, this point is (22,3π4,6)\ans{\left(2\sqrt{2},\dfrac{3π}{4},6\right)}(22,43π,6) in cylindrical coordinates. Spherical coordinates are similar to the way we describe a point on the surface of the earth using latitude and longitude. By specifying the radius of a sphere and the latitude and longitude of a point on the surface of that sphere, we can describe any point in R3.ℝ^3.R3. To describe the latitude and longitude, we use two angles: θθθ (the angle from the positive xxx axis) and ϕϕϕ (the angle from the positive zzz axis). We therefore have three coordinates (ρ,θ,ϕ),(ρ,θ,ϕ),(ρ,θ,ϕ), where ρρρ is the radius of the sphere. Note that 0≤θ